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A note on local isotropy criteria in shear flows with coherent motion

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Abstract

Whilst Local Isotropy (LI) is widely used, it is also necessary to test its validity, especially in shear flows, characterized by large-scale anisotropy. Important questions are whether the small scales are isotropic and how their properties depend on large-scale parameters (mean shear, the shear induced by a coherent motion, the Reynolds number etc.). We focus on two families of LI tests:
i) classical, kinematic tests, in which time-averages are compared to their isotropic values. The large-scale parameters do not appear explicitly. We only use here one example of such tests.
ii) Phenomenological tests, which explicitly account for the large-scale strain, as well as its associated dynamics. In flows populated by coherent motions in which phase-averages are pertinent for describing the flow dynamics, we propose a Local Isotropy (LI) criterion based on the intensity of the turbulent strain at a given scale $\vec{r}$ and a particular phase ϕ, $s(\vec{r},\phi)$. The formulation is the following: 'If LI were to be valid at a vectorial scale $\vec{r}$ and a phase ϕ, then the intensity of the turbulent strain $s_\phi(\vec{r},\phi)$ should prevail over the combined effect of the mean shear $\overline{S}$ and of the shear $\widetilde{S}$' associated with the coherent motion. The mathematical expression of $s(\vec{r},\phi)$ depends on the Laplacian of the total kinetic energy second-order structure function. Therefore, the proposed expression allows the eventual anisotropy to be taken into account. The new LI criterion is used together with data taken in the intermediate wake behind a circular cylinder. It is highlighted that *(i)* when $\widetilde{S}_\phi$ is important, LI only holds for scales smaller than the Taylor microscale *(ii)* when $\widetilde{S}_\phi$ is small, the domain in which LI is valid extends up to the largest scales.

Introduction

Local isotropy (LI) is seemingly one of the most important hypotheses on small-scale statistics. LI was first enunciated by Kolmogorov in 1941 [5], and further utilized and sometimes tested, in most of the laboratory flows. From the analytical viewpoint, LI leads to simplified expressions of e.g. the total kinetic energy, the dissipation rate of kinetic energy or scalar variance, structure functions at a given scale. Simple expressions of statistics are useful for the experimentalists, because of the limited possibilities to measure all the velocity components, as well as their spatial distribution.

Whilst LI is massively used, it is also necessary to test its validity, especially in shear flows, characterized by large-scale anisotropy. Important questions are whether the small scales are isotropic and which is the clear dependence of their statistics on large-scale parameters (mean shear $\overline{S}$, the shear induced by a coherent motion $\widetilde{S}$, the Reynolds number etc.).

Using a compilation of experimental and numerical data, Schumacher *et al.* [9] showed that LI prevails for small values of the ratio $\overline{S}/R_\lambda$ (R_λ is the Taylor microscale Reynolds number). One should expect that the magnitude of the shear will play some role in determining how high an R_λ is required for LI to prevail. Whereas the conclusion in [9] is optimistic quid the restoration of LI, the analytical study of [2] demonstrated that small scales cannot be isotropic in shear flows, independently of the values of R_λ and $\overline{S}$. From a general viewpoint, the assessment of LI can only be done through specific criteria and a definitive conclusion about the validity of LI is unlikely to be realistic.

The aim of this study is to understand how, in the context of shear flows, the anisotropy propagates across the scales from the largest to the smallest, how it evolves down the scales and finally, what the degree of anisotropy is at any given scale. To this end, we propose a phenomenological LI criterion based on the intensity of the turbulent strain at a given scale r.

As a first step in answering the question of what the isotropy level is at any particular scale, we consider flows populated by a single-scale, persistent coherent motion (hereafter, CM). A good candidate is the cylinder wake flow, and this study focuses entirely on this flow. The other advantage of investigating the wake flow is that it allows to invoke phase averages. The latter operation results in a dependence of any statistical quantities on the phase ϕ characterizing the temporal dynamics of the CM.

We focus on two families of LI tests:
i) classical, kinematic tests, in which time-averages are compared to their isotropic values. The large-scale parameters (shear) does not appear explicitly. We only use here one example of such tests.
ii) Phenomenological tests, which explicitly account for the large-scale strain, as well as its associated dynamics.

Phenomenological LI tests. Analytical considerations

The formulation of the LI criterion is the following: "For LI to be valid at a vectorial scale $\vec{r}$, then the intensity of the strain at that scale due to any larger scale must be much larger than the combined effect of the mean shear $\overline{S}$ and of the coherent motion shear, $\widetilde{S}$".

Mathematically, this can be expressed in terms of the following inequality

$$s_\phi(\vec{r},\phi) \gg \widetilde{S}_\phi, \tag{1}$$

with

$$\widetilde{S}_\phi = |\langle S\rangle|, \tag{2}$$

where $\langle . \rangle$ denotes phase averaging, $|\langle S\rangle| = \left|\overline{S}+\widetilde{S}\right|$ is the absolute value of the phase averaged strain $S = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)$ and $s_\phi(\vec{r},\phi)$ is the phase-averaged strain intensity at the scale $\vec{r}$ and the phase ϕ.

By integrating over all values of ϕ, the classical time averaged quantities are obtained. Under these conditions, the latter inequality reads

$$s(\vec{r}) \gg \widetilde{S}_t, \tag{3}$$

where $s(\vec{r})$ is the time-averaged strain intensity at the scale $\vec{r}$, and $S_t = \overline{\widetilde{S}_\phi}$.

The next step is to propose adequate expressions for the turbulent strain intensity $s(\vec{r})$ and $s_\phi(\vec{r},\phi)$. Starting from the definition of the strain tensor $\Sigma = \nabla_{\vec{x}}\vec{u}$, we need to further define the tensor $\mathcal{S}\Sigma$ characterizing the strain at a scale $\vec{r}$ associated with all the larger scales ([7], [1]), *i.e.* the quantity $\mathcal{S}\Sigma(\vec{r}) \equiv \nabla_{\vec{x}^+}\vec{u}^+ + \nabla_{\vec{x}}\vec{u}$, with $\vec{x}^+ = \vec{x}+\vec{r}$. By considering the two frames to be independent ([6]), with $\overline{\vec{U}}$ identical in the two frames and by invoking the same decomposition as proposed by e.g. [4] the final result is

$$\mathcal{S}\Sigma(\vec{r}) = \nabla_{\vec{r}}\Delta\vec{u}. \tag{4}$$

Therefore, as far as the turbulent field is concerned, the turbulent strain intensity, which is the norm of $\mathcal{S}\Sigma$, may be defined as follows

$$s(\vec{r}) = \overline{(\nabla_{\vec{r}}\Delta\vec{u})^2}^{1/2}. \tag{5}$$

After some calculations and by supposing that $\overline{\Delta u_j \frac{\partial^2}{\partial r_k^2}\Delta u_j} \approx 0$ (which is strictly true for $\vec{r} \to 0$), the final expression of $s(\vec{r})$ for turbulent flows in which time-averages are adequate, is the following

$$s(\vec{r}) = \left(\frac{1}{2}\mathcal{L}\overline{(\Delta u_i)^2}\right)^{1/2}(\vec{r}), \tag{6}$$

where repeated indices indicate summation and $\mathcal{L}$ represents the Laplacian operator. In flows populated by CM, in which phase-averages are more useful, the intensity of the strain depends on both $\vec{r}$ and the phase ϕ, and it reads

$$s_\phi(\vec{r},\phi) = \left(\frac{1}{2}\mathcal{L}\left\langle(\Delta u_i)^2\right\rangle\right)^{1/2}(\vec{r},\phi). \tag{7}$$

Calculating the Laplacian of these functions requires estimates of the velocity field in several planes, such as provided by PIV (Particle Image Velocimetry), or, preferably, numerical simulations.

It is important to note that, for LI, the Laplacian can be expressed in spherical coordinates as follows

$$s_\phi(r,\phi) = \left(\frac{1}{r}\frac{\partial}{\partial r}\left\langle(\Delta u_i)^2\right\rangle + \frac{1}{2}\frac{\partial^2}{\partial r^2}\left\langle(\Delta u_i)^2\right\rangle\right)^{1/2}(r,\phi). \tag{8}$$

The first term on the right side of Eq. (8) has already been proposed by [1]. This expression will be used later in this paper in order to infer s_ϕ and investigate phenomenological LI tests involving phase averages.

Experiments

Measurements were performed at the CORIA, University of Rouen, in a circular cylinder wake. The wind tunnel is of the recirculating type with a residual turbulence level smaller than 0.2 %. The test section is $0.4\times0.4\ m^2$ and 2.5m long and the mean pressure gradient was adjusted to zero. The circular cylinder of diameter $d = 10mm$ was placed horizontally, downstream the contraction, spanning the whole test section. The upstream velocity was $U_0 = 6.5m.s^{-1}$ corresponding to a Reynolds number based on the cylinder diameter of 4333 and a Taylor microscale Reynolds number of 70. Measurements were made at $70d$ downstream of the cylinder and for transverse positions varying between $y = 0$ and $y = 5d$.

Only the streamwise and the transverse velocity component u and v were measured. The X-wire probe (Dantec 55P51) was calibrated using a look-up table technique, with velocity increments of 1 m/s and angle increments of 5^0. The hot wires were operated by a Dantec constant temperature bridge, with an overheat ratio of 0.6. Voltage signals were passed through gain circuits (SRS SIM983) and low pass filtered (SRS SIM965) at a frequency close to the Kolmogorov frequency. The air temperature in the wind tunnel is kept constant during calibration and measurements, thus avoiding any systematic errors which may arise from slight variations in the mean temperature on the output characteristics of the hot wires.

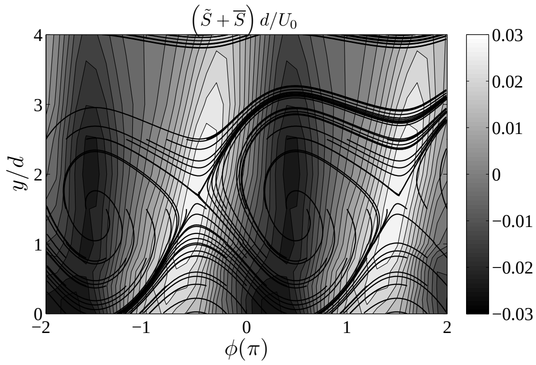

Figure 1: The total strain $\langle S\rangle \cdot d/U_0$ as a function of the phase ϕ and the vertical position in the wake, y/d.

Phase-averaged statistics are obtained as follows. The transverse velocity component v is first digitally band-pass filtered at the Strouhal frequency, using an eighth-order Butterworth filter. The filtering operation is applied to the Fourier transform of v in order to avoid any phase shift. Then, the Hilbert transform h of the filtered signal v_f is obtained and the phase ϕ inferred from the relation $\phi = \arctan\left(\frac{h}{v_f}\right)$. Finally, the phase is divided into 41 segments and phase-averaged statistics are calculated for each class. The convergence of statistics was checked, by reducing the number of classes, and found to be satisfactory. By means of our method, phase-averaged quantities are calculated over the period $[-\pi,\pi]$. As was done by [8], the phase is doubled up to $[-2\pi,2\pi]$ thanks to the periodicity, in order to enhance the visual display.

In [3], the geometrical space (location $\vec{x}$ in the flow) and the separation space (turbulent scales $\vec{r}$) are made independent by considering the geometrical location specified by the midpoint $\vec{X} = \frac{1}{2}\left(\vec{x}+\vec{x^+}\right)$ with $\vec{x^+} = \vec{x}+\vec{r}$. The same idea is applied here to phase-conditioned structure functions for which the phase ϕ is defined as the phase at the midpoint $\phi = \phi(\vec{X})$. Therefore, each velocity component is decomposed into a triple contribution from the mean temporal average, the phase-averaged fluctuation and the random/turbulent fluctuation.

Results in the wake flow

One-point statistics

One of the main advantages of using phase averages is that the temporal dynamics associated with the presence of the CM is highlighted. As far as the wake flow is concerned, one generally displays statistics in the (ϕ, y) plane to relate the spatial organization of the kinetic energy with that of the coherent structures. Here, we focus particularly on the coherent strain. The maxima of the total strain are noted at phases which are $-\pi/2+2n\pi$ (n is a positive integer), corresponding to the position of the saddle points (Fig. 1). Also illustrated in the same figure are the streamlines of the coherent vortices. The most visible are two of them, rotating clockwise (the upper stream moves from left to right). The centres of these vortices are located at $y/d \approx 1.4$ and phases $\phi = -3\pi/2 \pm 2n\pi$ and correspond to the minimum total strain $\langle S\rangle$. Note that on the wake centerline, the periodicity of the total strain is π, whereas it is 2π out of the centerline.

Phase-averaging inescapably leads to a dependence on the phase ϕ, and eventually on the scale r of the flow, as is the case for structure functions at any order.

Two-point statistics

Figure 2 represents the phase-averaged second-order structure functions for v normalized by its variance, $\langle(\Delta v)^2\rangle/\overline{v^2}$, as a function of the scale r/λ_u and the phase $\phi(\pi)$ of the coherent motion.

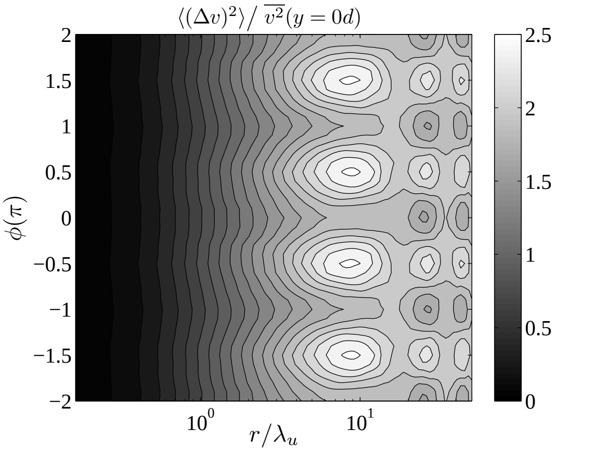

Figure 2: Values of $\langle(\Delta v)^2\rangle(r,\phi)/\overline{v^2}$ as a function of r/λ_u and the phase $\phi(\pi)$ at $y=0d$ and $R_{\lambda_u}\simeq 70$.

The values of the scale-phase second-order structure functions progressively increase as r keeps increasing, and reach a maximum for $r/\lambda_u \approx 9$ (this scale is equal to half the distance between two successive vortices), followed first by a slight decrease and then by a quasi-periodic behaviour for the largest scales. The maxima of $\langle(\Delta v)^2\rangle(r,\phi)/\overline{v^2}$ occur at scales which are multiples of the first maximum. The trend of $\langle(\Delta v)^2\rangle(r,\phi)/\overline{v^2}$ is uniform in ϕ for small scales, consistent with the fact that only turbulent fluctuations are present at these scales. At larger scales, where the CM is present, there is a hint of periodicity along the ϕ axis (for $r/\lambda_u \approx 2$), followed by a clear periodicity at scales $r/\lambda_u \approx 9$, as emphasised earlier. At these scales, turbulent fluctuations diminish and the CM is predominant.

The dynamical aspect of $\langle(\Delta v)^2\rangle(r,\phi)$ should be understood in association with the phase variations of the total strain, Fig. 1. A careful analysis of this figure reveals that the extrema of the total strain $\overline{S}+\widetilde{S}$ occur at the same phases (i.e. odd multiples of $\pi/2$) almost independently of the spatial location y.

Local Isotropy criterion

LI is first assessed from a kinematic LI test relating phase-conditioned second-order structure functions. In this context, the isotropic relation between second-order structure functions of the longitudinal velocity components and those of the transverse velocity components may be written as

$$\left\langle(\Delta u_\perp)^2\right\rangle_{iso}(r,\phi)=\left(1+\frac{r}{2}\frac{\partial}{\partial r}\right)\left\langle(\Delta u)^2\right\rangle(r,\phi), \qquad (9)$$

Note the analogy between (9) and its time-averaged counterpart

$$\overline{(\Delta u_\perp)^2}_{iso}(r)=\left(1+\frac{r}{2}\frac{\partial}{\partial r}\right)\overline{(\Delta u)^2}, \qquad (10)$$

It is obvious that such a LI criterion at each phase of the motion is much more restrictive than its time-averaged counterpart, Eq. (10).

We present results for the ratio $\left\langle(\Delta u_\perp)^2\right\rangle_{iso}(r,\phi)/\left\langle(\Delta u_\perp)^2\right\rangle(r,\phi)$, where $\left\langle(\Delta u_\perp)^2\right\rangle_{iso}$ is given by relation (9). This ratio is illustrated in Fig. 3 on the wake centerline.

The most important remark concerns the positions at which the maximum departure of the ratio from the isotropic value of 1 is observed. There are points for which the value of $\left\langle(\Delta v)^2\right\rangle_{iso}(r,\phi)/\left\langle(\Delta v)^2\right\rangle(r,\phi)$ is 0.8. This occurs at phases which are odd multiples of $\pi/2$, for which the absolute value of the total strain is maximal, and with scales as large as $\approx 8\lambda_u$. Therefore, by considering the phase-conditioned LI test Eq. 9, the relation between anisotropy and the coherent strain can be emphasized.

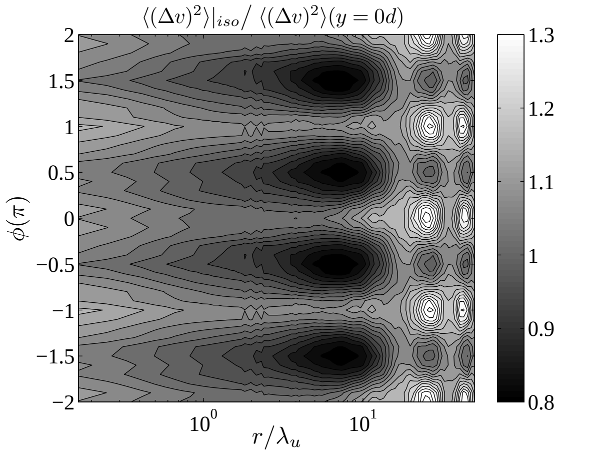

Figure 3: The dependence of $\left\langle(\Delta v)^2\right\rangle_{iso}(r,\phi)/\left\langle(\Delta v)^2\right\rangle(r,\phi)$ on r/λ_u and $\phi(\pi)$ at $y/d=0$ and $R_{\lambda_u}\simeq 70$.

We now turn our attention to the phenomenological LI criterion proposed in this study and thus $s_\phi(r,\phi)\gg\tilde{S}_\phi(\phi)$ is tested against experimental data. Figure 4 depicts $\log_{10}(\tilde{S}_\phi/s_\phi)$ as functions of r/λ_u and the phase ϕ/π at a spatial location $y/d=0$ on the centerline. A possible statement of the LI criterion is 'LI should hold if $\log_{10}(\tilde{S}_\phi/s_\phi)\le -1$'. Small values of $\log_{10}(\tilde{S}_\phi/s_\phi)$ (dark zones) occur for small scales, whereas large values (much larger than 10^{-1}), as highlighted by white regions, are found mostly at large scales. The curve for which $\log_{10}(\tilde{S}_\phi/s_\phi)=-1$, i.e. '$L_{-1}$', is represented by dotted lines. This curve separates the region of small values of r (for which LI holds), from the region of large anisotropic scales. As emphasised by this figure, L_{-1} varies between $0.8\lambda_u$ and $8\lambda_u$. The phase for which L_{-1} is minimum is fully correlated with the extremum values of the coherent strain $\tilde{S}_\phi$ (Fig. 1) and of the maximum of anisotropy. At the phases for which $\tilde{S}_\phi=0$, the influence of the coherent motion is absent, so that LI becomes more noticeable and L_{-1} can increase.

Conclusion

An original LI test based on the ratio between the intensity of the turbulent strain and that of the combined effect of the mean and coherent shear is proposed. This test is phenomenological and thus has an explicit dependence on the total large scale strain which induces anisotropy.

It has been shown that *(i)* when $\tilde{S}_\phi$ is important, LI only holds for scales smaller than the Taylor microscale *(ii)* when $\tilde{S}_\phi$ is

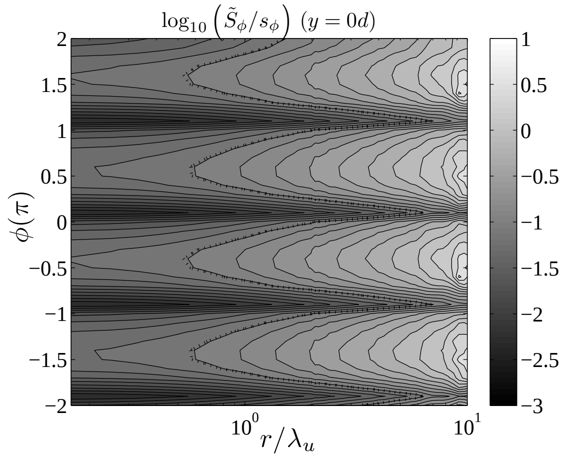

Figure 4: Values of $\log_{10}(\tilde{S}_\phi/s_\phi)$ as functions of r/λ_u and the phase ϕ/π, at $y/d=0$. The dotted lines represent the scale L_{-1}.

small, the domain in which LI is valid extends up to the largest scales.

The analytical tool we have developed opens perspectives for a better understanding of the validity of LI in both decaying and shear flows, as a function of the dynamical behavior of large-scale statistics.

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